Construction of Velocity and Acceleration Diagrams

**In a Nut Shell:** The relative velocity and relative acceleration equations shown below lend themselves to representation using vector diagrams. The vector on the left hand side of the equal sign must equal the sum of the vectors on the right hand side. Plots of these vector diagrams aid in understanding the physical nature of each term.

**Summary:**

<table>
<thead>
<tr>
<th>Relative Velocity Equation</th>
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<tbody>
<tr>
<td>( \mathbf{v}_C = \mathbf{v}<em>A + \mathbf{v}</em>{CA} = \mathbf{v}<em>A + \mathbf{\omega} \times \mathbf{r}</em>{AC} )</td>
</tr>
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<td>( \mathbf{a}_C = \mathbf{a}<em>A + \mathbf{a}</em>{CA</td>
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<td>( \mathbf{a}<em>C = \mathbf{a}<em>A + \mathbf{\omega} \times \mathbf{\omega} \times \mathbf{r}</em>{AC} + \mathbf{\alpha} \times \mathbf{r}</em>{AC} )</td>
</tr>
<tr>
<td>( \mathbf{a}<em>C = \mathbf{a}<em>A - \mathbf{\omega}^2 \mathbf{r}</em>{AC} + \mathbf{\alpha} \times \mathbf{r}</em>{AC} )</td>
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**Meaning of terms**  
**Note:** Both points, A and C, are in the same rigid link.

- \( \mathbf{v}_C \) is velocity of point C with respect to the fixed frame F
- \( \mathbf{v}_A \) is velocity of point A with respect to the fixed frame F
- \( \mathbf{v}_{CA} \) is the velocity of point C with respect to point A in the fixed frame F
- \( \mathbf{\omega} \) is the angular velocity of link B in the fixed frame F
- \( \mathbf{r}_{AC} \) is the position vector from A to C
- \( \mathbf{a}_C \) is the acceleration of point C with respect to the fixed frame F
- \( \mathbf{a}_A \) is the acceleration of point A with respect to the fixed frame F
- \( \mathbf{a}_{CA|n} \) is the normal component of the acceleration of point A with respect to point C in the fixed frame F
- \( \mathbf{a}_{CA|t} \) is the tangential component of the acceleration of point A with respect to point C in the fixed frame F
- \( \mathbf{\alpha} \) is the angular acceleration of link B in the fixed frame F
- \( \mathbf{\alpha} \times \mathbf{r}_{AC} \) is the tangential acceleration of C relative to A in frame F
- \( -\mathbf{\omega}^2 \mathbf{r}_{AC} \) is the normal acceleration of C relative to A in the fixed frame F
- \( \mathbf{\omega} \) is the angular speed of link B in the fixed frame F
**Construction of the Velocity Vector**

**Given:** The velocity of point A, $v_A$, $r_{AC}$, and $\omega \mathbf{k}$ on link AC

**Objective:** Draw vector diagram for $v_C$.
Note: The velocity of C relative to A, $V_{CA}$, is perpendicular to the line from A to C (Fig 2A).

It's what you see if you sit at A and look at C. The direction comes from the sign of the angular velocity, $\omega$, (Fig 2B) and has a magnitude equal to the distance, AC, times the angular speed, $\omega$ (Fig 2C).

Add vectors on RHS to obtain vector representation of $V_C$ on LHS

i.e. $V_C = V_A + V_{CA}$

Also $V_C = V_A + \omega \times r_{AC}$

Construction of the Acceleration Vector

Given: $a_A$, $r_{AC}$, $\omega k$, $\alpha k$ on link AC

Objective: Draw vector diagram for $a_C$.

![Diagram of rigid link with vectors $V_A$, $V_{CA}$, $a_A$, $\omega k$, $\alpha k$ and $F$ as a fixed frame of reference.]}
Note: The normal component of relative acceleration, $a_{C/A\parallel}$, is directed from C to A (Fig 2A, 2B).

Its magnitude is $\lvert AC \rvert \omega^2$ (Fig 2C). The tangential component of relative acceleration, $a_{C/A\perp}$ is normal to the line from C to A (Fig 2A,2B). Its direction comes from the sign of the angular acceleration (Fig. 2C). Its magnitude is $\lvert AC \rvert \alpha$ (Fig 2C).

Add vectors on RHS to obtain vector diagram for $a_C$ on LHS.

$$a_C = a_A + a_{C/A\parallel} + a_{C/A\perp}$$

$$a_C = a_A + \omega \times \omega \times r_{AC} + \alpha \times r_{AC}$$

$$a_C = a_A - \omega^2 r_{AC} + \alpha \times r_{AC}$$